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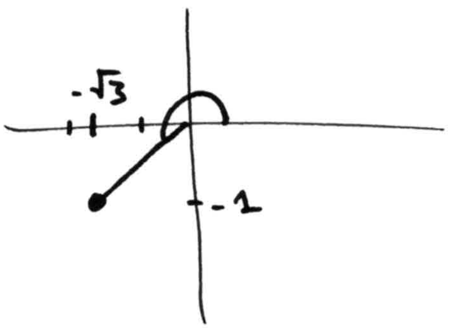
Polar Coordinates, Equations, and Area

1. Find a polar coordinate representation of the point with Cartesian (rectangular) coordinates $(-\sqrt{3}, -1)$.



know $r = \sqrt{(-\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

know $\tan\theta = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$

inside $\measuredangle$ is $\pi/6$

Pick $\theta = \frac{7\pi}{6}$

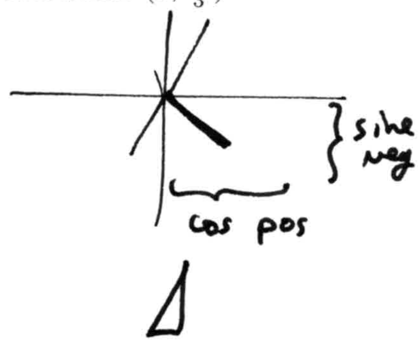
$(r,\theta) = \left(2, \frac{7\pi}{6}\right)$

2. Find the Cartesian (rectangular) coordinates for the point with polar coordinate $(3, \frac{5\pi}{3})$

$x = r\cdot\cos\theta = 3\cdot\cos\left(\frac{5\pi}{3}\right) = 3\left(+\frac{1}{2}\right)$

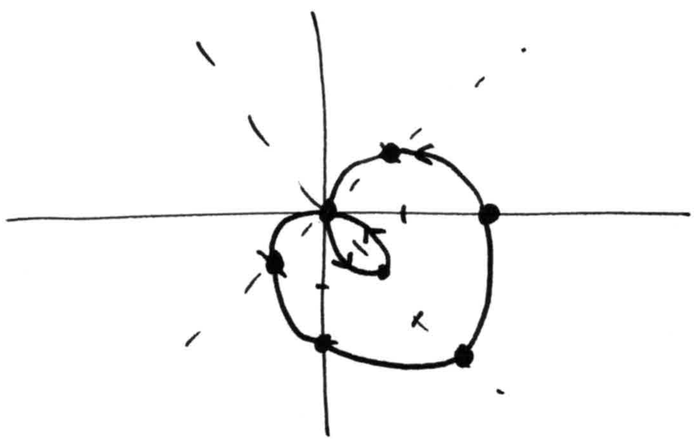
$y = r\cdot\sin\theta = 3\cdot\sin\left(\frac{5\pi}{3}\right) = 3\cdot\left(-\frac{\sqrt{3}}{2}\right)$

$(x,y) = \left(\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right)$



3. Sketch the polar curve corresponding to the table below (assume that r is continuous and smooth):

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	2	1	0	-1	0	1	2	3	2



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want $r = f(\theta)$

4. Obtain a polar function which graphs the Cartesian equation

$$xy^2 = 1$$

$$(r\cdot\cos\theta)(r\cdot\sin\theta)^2 = 1$$

$$r\cdot\cos\theta\cdot r^2\cdot\sin^2\theta = 1$$

$$r^3 = \frac{1}{\cos\theta\cdot\sin^2\theta}$$

$$r = \sqrt[3]{\frac{1}{\cos\theta\cdot\sin^2\theta}}$$

know

$r^2 = x^2 + y^2$

$x = r\cdot\cos\theta$

$y = r\cdot\sin\theta$

want: an equation with ONLY x's and y's (NO r's & θ's)

5. Find a Cartesian equation for the graph of the polar function

$$r = \sin^2(\theta)$$

$\boxed{\text{trick 1 rewrite to "find" } x \& y \text{ in the eqn}}$

$$r^2\cdot r = r^2\cdot\sin^2\theta$$

$$r^3 = (r\cdot\sin\theta)^2$$

$$\left(\sqrt{x^2+y^2}\right)^3 = y^2$$

know

$r^2 = x^2 + y^2$

$r = \sqrt{x^2+y^2}$

$x = r\cdot\cos\theta$

$y = r\cdot\sin\theta$

OR

trick 2: get rid of r, $\sin\theta$, & $\cos\theta$

$$\begin{cases} r = \sqrt{x^2+y^2} \\ \sin\theta = y/r \\ \cos\theta = x/r \end{cases}$$

$$r = \sin^2\theta$$

$$\sqrt{x^2+y^2} = \left(\frac{y}{r}\right)^2$$

$$\sqrt{x^2+y^2} = \frac{y^2}{\left(\sqrt{x^2+y^2}\right)^2}$$

$$\left(\sqrt{x^2+y^2}\right)^3 = y^2$$

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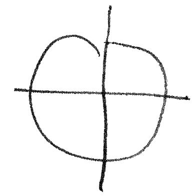
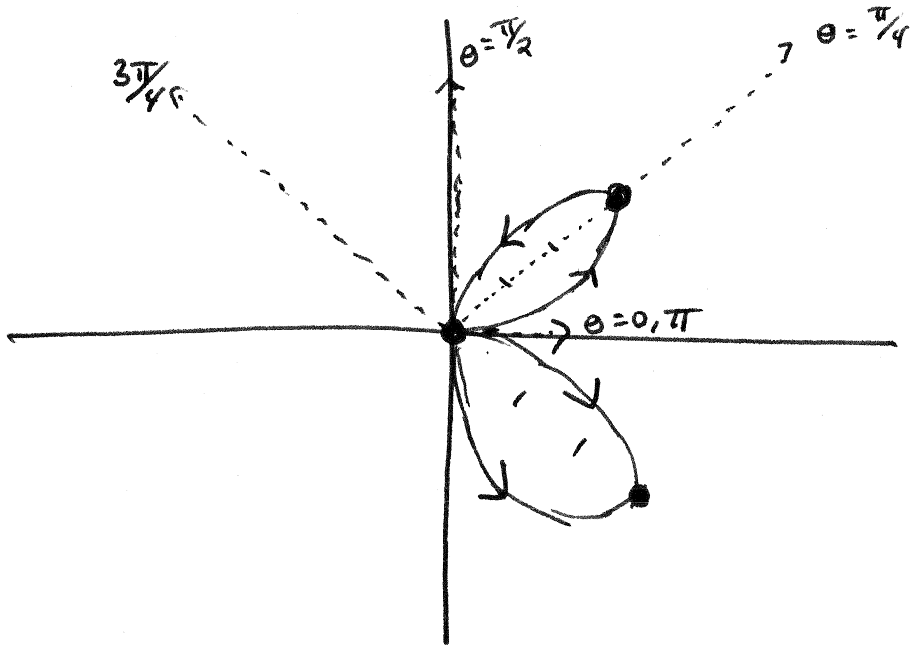


6. Fill in the following table of values for the function $r = 3\sin(2\theta)$

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
r	$3\cdot\sin(0)$ $= 0$	$3\cdot\sin(2\cdot\frac{\pi}{4})$ $=3\cdot\sin(\pi/2)$ $= 3$	$3\cdot\sin(2\cdot\frac{\pi}{2})$ $=3\cdot\sin(\pi)$ $=0$	$3\cdot\sin(2\cdot\frac{3\pi}{4})$ $=3\cdot\sin(\frac{3\pi}{2})$ $=-3$	$3\cdot\sin(2\pi)$ $=3\cdot 0$ $=0$

Use it to sketch the ***restriction*** of the polar curve to θ in $[0, \pi]$.



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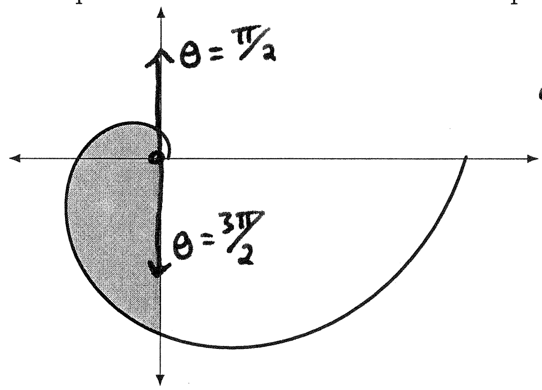
7. Set up and calculate the area under a spiral $r = \theta^2 + 1$ sketched below.



$$\text{area} = \int_{\pi/2}^{3\pi/2} \frac{1}{2}(\theta^2+1)^2\, d\theta$$

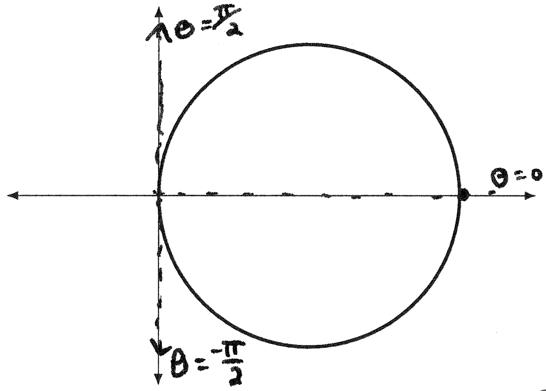
$$= \int_{\pi/2}^{3\pi/2} \frac{1}{2}(\theta^4+2\theta^2+1)\, d\theta$$

$$= \left[\frac{1}{2}\left(\frac{\theta^5}{5} + \frac{2\theta^3}{3} + \theta\right)\right]_{\pi/2}^{3\pi/2}$$

$$= \frac{1}{2}\left(\frac{\left(\frac{3\pi}{2}\right)^5}{5} + \frac{2\left(\frac{3\pi}{2}\right)^3}{3} + \left(\frac{3\pi}{2}\right)\right) - \frac{1}{2}\left(\frac{\left(\frac{\pi}{2}\right)^5}{5} + \frac{2\left(\frac{\pi}{2}\right)^3}{3} + \frac{\pi}{2}\right)$$

DONE!

8. Calculate the area of the circle $r = 4\cos(\theta)$ as an integral in polar coordinates. Be careful to choose the correct limits of integration.



$$\text{area} = \int_{-\pi/2}^{\pi/2} \frac{1}{2}(4\cdot\cos\theta)^2\, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \frac{1}{\cancel{2}}\cdot \cancel{16}\cdot\cos^2\theta\, d\theta$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\frac{d}{d\theta}\left[\frac{\sin(2\theta)}{2}\right] = \frac{\cos(2\theta)\cdot\cancel{2}}{\cancel{2}}$$

antiderivative of 1

antiderivative of cos(2θ

$$= \int_{-\pi/2}^{\pi/2} \overset{4}{\cancel{8}}\cdot\left(\frac{1+\cos(2\theta)}{\cancel{2}}\right) d\theta = \left[4\left(\theta + \frac{\sin(2\theta)}{2}\right)\right]_{-\pi/2}^{\pi/2}$$

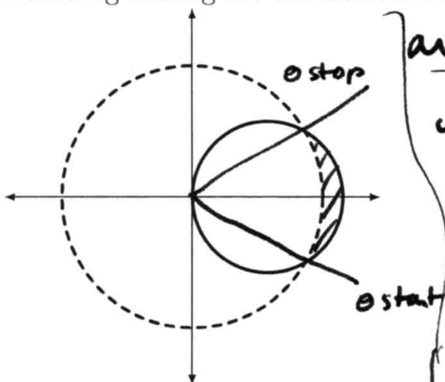
$$= 4\left(\frac{\pi}{2} + \frac{\cos\left(2\frac{\pi}{2}\right)}{2}\right) - 4\left(\frac{-\pi}{2} + \frac{\cos\left(2\frac{-\pi}{2}\right)}{2}\right) = 2\pi + \cancel{(-2)} - (-2\pi) - \cancel{2(-1)}$$

$$= 4\pi$$

$\cos(\pi) = -1 \qquad \cos(-\pi) = -1$

9. Sketch the area of the region inside the curve $\overset{r_{outer}}{r = 2\cos(\theta)}$ and outside the circle $\overset{r_{inner}}{r = \sqrt{3}}$.

   Then (1) carefuly select your limits of integration and (2) evaluate the integral up to and including finding the antiderivative. **Do not** evaluate the antiderivative numerically.



area starts when curves intersect

when $2\cdot\cos(\theta) = \sqrt{3}$

$\cos(\theta) = \sqrt{3}/2$

when inside & $\theta = \pi/6$

θ start $= -\pi/6$, θ stop $= \pi/6$

Area between = Area inside outer − Area inside inner

$$= \int_{-\pi/6}^{\pi/6} \frac{1}{2}(2\cdot\cos\theta)^2\,d\theta - \int_{-\pi/6}^{\pi/6} \frac{1}{2}(\sqrt{3})^2\,d\theta$$

limits are same ⇒ can combine integrals

$$= \int_{-\pi/6}^{\pi/6} \underbrace{\cancel{\tfrac{1}{2}}\cdot\cancel{4}}_{2}\cdot\cos^2\theta - \frac{1}{2}\cdot 3\cdot d\theta$$

Know $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

$$= \int_{-\pi/6}^{\pi/6} \cancel{2}\cdot\frac{1+\cos 2\theta}{\cancel{2}} - \frac{3}{2}\,d\theta$$

$$= \int_{-\pi/6}^{\pi/6} 1 + \cos 2\theta - \frac{3}{2}\,d\theta$$

Know $\dfrac{d}{d\theta}\left[\dfrac{\sin(2\theta)}{2}\right] = \dfrac{\cos(2\theta)\cdot\cancel{2}}{\cancel{2}}$

$$= \left[\theta + \frac{\sin(2\theta)}{2} - \frac{3}{2}\theta\right]_{\theta=-\pi/6}^{\theta=\pi/6}$$

required here.

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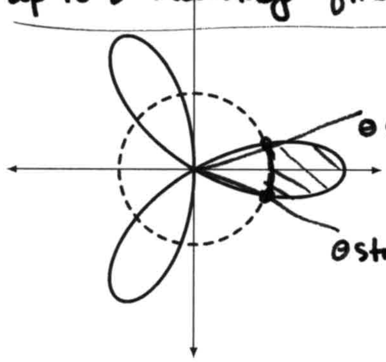
10. Sketch the area of the region inside one leaf of the rose $r = \cos(3\theta)$ and outside the circle $r = \frac{1}{2}$.

(1) Set up and (2) Compute the integral, being careful to select your limits of integration.

up to & including finding the antiderivative.

area starts & stops when curves intersect



when $\cos(3\theta) = \frac{1}{2}$

when $3\theta = \pm\frac{\pi}{3}$

when $\theta = \pm\frac{\pi}{9}$

Area Between = Area inside outer − Area inside inner

$$= \int_{-\pi/9}^{\pi/9} \frac{1}{2}\left(\cos(3\theta)\right)^2 d\theta - \int_{-\pi/9}^{\pi/9} \frac{1}{2}\left(\frac{1}{2}\right)^2 d\theta$$

same limits $\Rightarrow$ can combine integrals

$$= \int_{-\pi/9}^{\pi/9} \frac{1}{2}\cos^2(3\theta) - \frac{1}{8}\, d\theta$$

know: $\cos^2 x = \frac{1+\cos 2x}{2}$

$\cos^2(3\theta) = \frac{1+\cos(6\theta)}{2}$

$$= \int_{-\pi/9}^{\pi/9} \frac{1}{2}\cdot\frac{1+\cos(6\theta)}{2} - \frac{1}{8}\, d\theta$$

$$= \int_{-\pi/9}^{\pi/9} \frac{1}{4} + \frac{1}{4}\cos(6\theta) - \frac{1}{8}\, d\theta$$

know

$$\frac{d}{d\theta}\left[\frac{\sin(6\theta)}{6}\right] = \frac{\cos(6\theta)\cdot\cancel{6}}{\cancel{6}}$$

$$= \boxed{\left[\frac{1}{4}\theta + \frac{1}{4}\cdot\frac{\sin(6\theta)}{6} - \frac{1}{8}\theta\right]_{-\pi/9}^{\pi/9}}$$

must specify here.